

SHORT COMMUNICATIONS

Analysis of melting driven by friction and temperature difference under sliding plate*

CHEN Wenzhen**, ZHAO Yuansong, SUN Fengrui, ZHU Bo and GONG Miao

(College of Power and Shipping, Naval University of Engineering, Wuhan 430033, China)

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Abstract The contact melting processes of phase change material (PCM) under a sliding plate, which is driven by the friction and temperature difference ΔT , are studied. By using film theory, the fundamental equations for the melting process are derived. The thickness of boundary layer, the pressure distribution inside boundary layer and the mass melting rate of PCM are also obtained with the numerical method. It is found that (1) the larger the temperature difference is, the larger the thickness of boundary layer and the mass melting rate are, and the more asymmetric the pressure distribution is; (2) the quicker the sliding velocity is, the larger the mass melting rate is, the flatter the thickness of boundary layer is, and the more symmetric the pressure distribution is; (3) the results of the contact melting driven together by the friction and temperature difference cannot be obtained by the addition of those driven respectively by the friction and temperature difference.

Keywords: friction, contact melting, sliding plate.

The contact melting of the solid phase change material (PCM) around a rigid body has been widely used in numerous natural and technological processes, such as ice-out, skating, metallurgy, lubrication and nuclear technology. The contact melting and lubrication are considered to be related to heat transfer, hydrodynamics, tribology and manufacturing^[1-3]. The classical application of melting lubrication is in the area of sliding friction on ice and snow. Among the more modern applications is the coating of a metallic part with another metal whose melting point is considerably lower. The function of the latter is to melt and serve as lubricant in a manufacturing process to which the former may be subjected. Another application is in the field of interior ballistics, where a projectile (e. g., brass bullet) melts superficially as it travels along a gun barrel, and so on. Over the past years, a large number of literatures^[4-12] have been published concerning contact melting driven by temperature difference ΔT and pressure difference Δp , and many useful results were obtained. Comparatively the study on the contact melting driven by friction^[13,14] is rare. In addition, after comparing the similarities and differences between temperature difference and friction melting, Morega et al.^[15] tried to directly obtain the results for contact melting due to

temperature difference by the substitution of the related parameter for contact melting due to friction, but they were defeated. In fact, although there are many analogies between temperature difference and friction melting, the results for them cannot be obtained by the simple substitution of the related parameter. As noted earlier Chen and Yang^[11] considered pressure contact melting around a horizontal cylinder and sphere. They concluded that though the relationship between the pressure and temperature difference is linearity, the results of the ΔT -driven contact melting cannot be obtained from that of Δp -driven contact melting by the substitution of equivalent temperature difference. In the present work, the contact melting processes of phase change material (PCM) under a sliding plate, which is driven together by the friction and temperature difference, are studied. Considering the mutual effect of the friction and temperature difference, we derive the fundamental equations for the melting process by the film theory. The liquid layer thickness, the pressure distribution and the mass melting rate are also obtained.

1 Theoretic analysis

Consider the contact melting of solid PCM under

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** To whom correspondence should be addressed. E-mail: cwz2@21cn.com

a sliding plate, which is driven together by the friction and temperature difference ΔT . The physical model and coordinates considered in this investigation are shown in Fig. 1. The plate with width W and constant temperature T_w as well as uniformity sliding velocity V is placed in and acted on the solid PCM by a down force F . The temperature of PCM is uniform, remaining at the melting point T_m , smaller than T_w . It is assumed that (1) the melted liquid layer is very thin (i. e., $\delta \ll W$) and the viscous forces are dominant; (2) heat transport by convective flow is negligible as compared with that by heat conduction in the quasi steady liquid layer. Other assumptions are the same as those used by Bejan and Fowler^[13,14]. Then we can obtain the liquid movement and continuum equations inside the boundary layer as follows, respectively,

$$\mu \frac{\partial^2 u}{\partial s^2} = \frac{dp}{dx}, \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial s} = 0. \quad (2)$$

The related boundary conditions are

$$\left. \begin{aligned} s = 0: u = V, v = 0 \\ s = \delta: u = 0, v = -U \end{aligned} \right\}, \quad (3)$$

where u and v are the velocity of molten liquid along x and s direction, respectively; p is the pressure inside the liquid layer; μ is the dynamic viscosity; U is the melting velocity of PCM. From Fig. 1 we can get $U = Vd\delta/dx$. Solving Eqs. (1) and (2) with the boundary conditions (3) results in

$$u = \frac{1}{2\mu} \frac{dp}{dx} s(s - \delta) + V(1 - s/\delta), \quad (4)$$

$$\frac{\delta^3}{\mu} \left(\frac{dp}{dx} \right) + 6V\delta = C, \quad (5)$$

where C is a constant to be determined. The force per unit length exerted on the sliding plate F is

$$F = \int_0^W p dx. \quad (6)$$

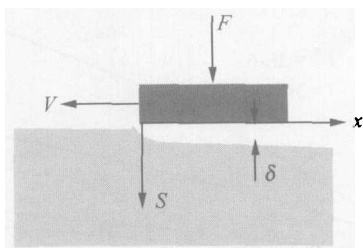


Fig. 1. Schematic of friction melting under a sliding plate.

The energy balance equation with the friction dissipation inside the liquid layer is written as follows:

$$\lambda \frac{\partial^2 T}{\partial s^2} + \mu \left(\frac{\partial u}{\partial s} \right)^2 = 0. \quad (7)$$

Substituting Eq. (4) into Eq. (7) and integrating it results in

$$\begin{aligned} T = & -\frac{\mu}{\lambda} \left[\frac{1}{4\mu^2} \left(\frac{dp}{dx} \right)^2 \left(\frac{1}{3}s^4 - \frac{2\delta}{3}s^3 + \frac{\delta^2}{2}s^2 \right) \right. \\ & + \frac{1}{2} \left(\frac{V}{\delta} \right)^2 s^2 - \frac{V}{\mu\delta} \frac{dp}{dx} \left(\frac{1}{3}s^3 - \frac{\delta}{2}s^2 \right) \Big] \\ & + D_1 s + D_2, \end{aligned} \quad (8)$$

where D_1 and D_2 are the constant to be determined by the temperature conditions respectively, i. e., $T = T_m$ while $s = \delta$ and $T = T_w$ while $s = 0$. Substituting the temperature conditions into Eq. (8) we get

$$\begin{aligned} T_m = & -\frac{\mu}{\lambda} \left[\frac{1}{24\mu^2} \left(\frac{dp}{dx} \right)^2 \delta^4 + \frac{1}{2} V^2 + \frac{V}{6\mu} \frac{dp}{dx} \delta^2 \right] \\ & + D_1 \delta + D_2, \\ T_w = & D_2. \end{aligned}$$

Obtaining D_1 and D_2 from above the two equations and substituting them into Eq. (8), the temperature distribution inside the liquid layer can be derived as follows:

$$\begin{aligned} T = & \frac{1}{2\lambda} \left[\frac{1}{12\mu} \left(\frac{dp}{dx} \right)^2 (\delta^3 s - 3\delta^2 s^2 + 4\delta s^3 - 2s^4) \right. \\ & + \mu \left(\frac{V}{\delta} \right)^2 (\delta s - s^2) + \frac{V}{3} \frac{dp}{dx} (\delta s - 3s^2 + \frac{2}{\delta} s^3) \Big] \\ & + (T_m - T_w) \frac{s}{\delta} + T_w. \end{aligned} \quad (9)$$

The energy balance equation between the solid-liquid surfaces of PCM is

$$-\lambda (\partial T / \partial s) |_{s=\delta} = \rho L V (d\delta / dx), \quad (10)$$

where λ , ρ and L are the thermal conductivity, density and melting latent heat of PCM, respectively.

Substituting Eq. (9) into Eq. (10) yields

$$\begin{aligned} \frac{\delta^4}{24\mu} \left(\frac{dp}{dx} \right)^2 + \frac{\mu V^2}{2} - \frac{V\delta^2}{6} \frac{dp}{dx} + (T_w - T_m) \lambda \\ = \delta \rho L V \frac{d\delta}{dx}. \end{aligned} \quad (11)$$

The dimensionless parameters are defined as follows:

$$\begin{aligned} \delta^* &= \frac{\delta}{W}; \quad x^* = \frac{x}{W}; \quad p^* = \frac{p}{F/W}; \\ V^* &= \frac{V\mu}{F}; \quad f = \frac{F}{\rho L W}; \quad t = \frac{\lambda \mu (T_w - T_m)}{F^2}. \end{aligned}$$

Then Eqs. (5), (6) and (11) can be rewritten as follows:

$$\delta^{*3} \left(\frac{dp^*}{dx^*} \right) + 6V^* \delta^* = C^*, \quad (12)$$

$$1 = \int_0^1 p^* dx^*, \quad (13)$$

$$\delta^{*4} \left(\frac{dp^*}{dx^*} \right)^2 + 12V^{*2} - 4V^* \delta^{*2} \frac{dp^*}{dx^*} + 24t - 24 \frac{V^*}{f} \delta^* \frac{d\delta^*}{dx^*} = 0, \quad (14)$$

where $C^* = 12\mu C/(FW)$ is a dimensionless constant to be determined. The mass melting rate of PCM during the sliding of the plate is

$$M = \left(\rho \int_0^w \int_0^\delta u ds dx \right) / W. \quad (15)$$

Substituting Eqs. (4) and (5) into Eq. (15), the dimensionless mass melting rate can be obtained as follows:

$$M^* = \frac{M}{\rho WV} = \int_0^1 \delta^* dx^* + \frac{C^*}{12V^*}. \quad (16)$$

Eqs. (12), (13), (14) and (16) are the fundamental equations which can be used to describe the contact melting driven together by the friction and temperature difference. With the boundary condition $p^* = 0$ while $x^* = 0$ and 1, the mass melting rate of PCM, the thickness of boundary layer and the pressure distribution inside boundary layer during the contact melting can be obtained from Eqs. (12), (13), (14) and (16).

2 Results and discussion

Here, *n*-octadecane is chosen as the PCM for calculation and discussion. The thermophysical property of the PCM is $T_m = 28^\circ\text{C}$, $L = 224 \text{ kJ/kg}$, $\rho = 814 \text{ kg/m}^3$, $\lambda = 0.358 \text{ W/(m}\cdot^\circ\text{C)}$, $\mu = 3.9 \times 10^{-3} \text{ kg/(m}\cdot\text{s)}$. Figs. 2–5 illustrate the variation curves of the dimensionless liquid layer thickness δ^* and pressure p^* obtained by the numerical method from Eqs. (12), (13) and (14). From Figs. 2 and 3 it is found that liquid layer thickness gradually increases along x during the sliding of the plate, which is analogous to the results obtained by Fowler and Bejan^[14] for the pressure and friction contact melting. From Figs. 2 and 3 as well as the definition of V^* and t , it is also seen that the variation curves of the liquid layer thickness will become flatter when the sliding velocity increases and temperature difference ($T_w - T_m$) decreases. While $t = 0$, namely the contact melting is due to the friction, the liquid layer thickness is a constant beeline, which is the same as the results for the contact melting driven by the tem-

perature difference^[16].

Figs. 4 and 5 show that the pressure distribution inside boundary layer is close to a parabola curve, but the maximum pressure is in the front of the plate along the sliding direction. From Fig. 4, it is seen that when the sliding velocity V increases the pressure distribution curve will move to the back along the sliding direction and the maximum pressure will decrease. Similarly, it is also seen from Fig. 5 that the variation law of the pressure distribution with the temperature difference ($T_w - T_m$) is opposite to that with the sliding velocity V , namely when the temperature difference ($T_w - T_m$) decreases the pressure distribution curve will move to the back along the sliding direction and the maximum pressure will decrease. But when the temperature difference ($T_w - T_m$) decreases to a fixed value, its effects on the pressure distribution and the maximum will disappear. While $t = 0$, namely the contact melting is due to the friction, the pressure distribution is a parabola, which is also the same as the results for the contact melting driven by the temperature difference^[16].

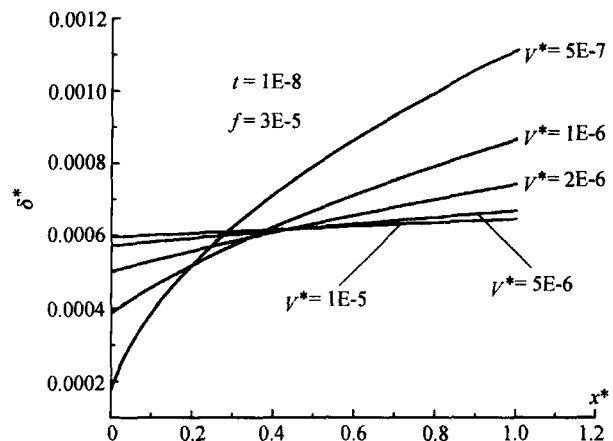


Fig. 2. Effect of sliding velocity on layer thickness.

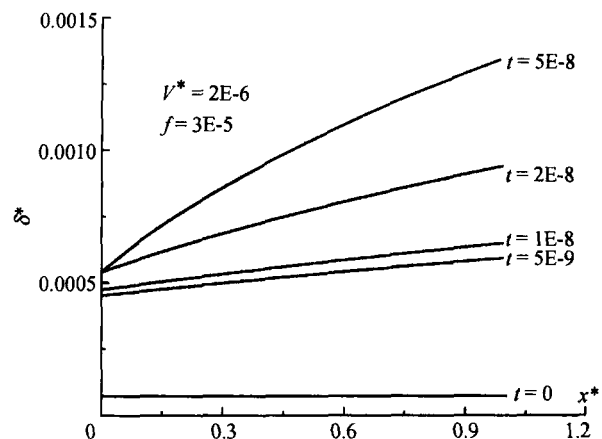


Fig. 3. Effect of temperature difference on layer thickness.

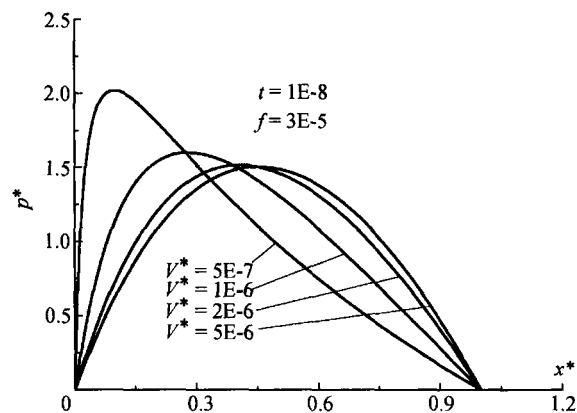


Fig. 4. Effect of sliding velocity on pressure distribution.

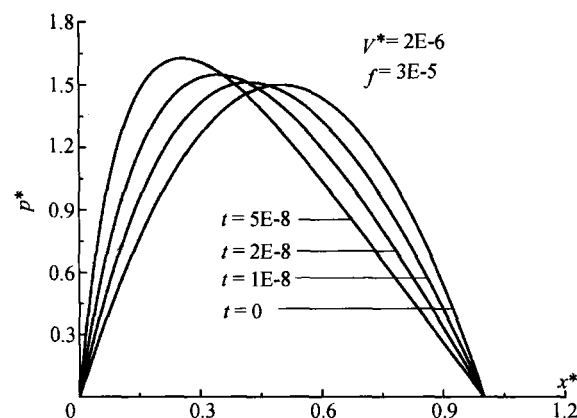


Fig. 5. Effect of temperature difference on pressure distribution.

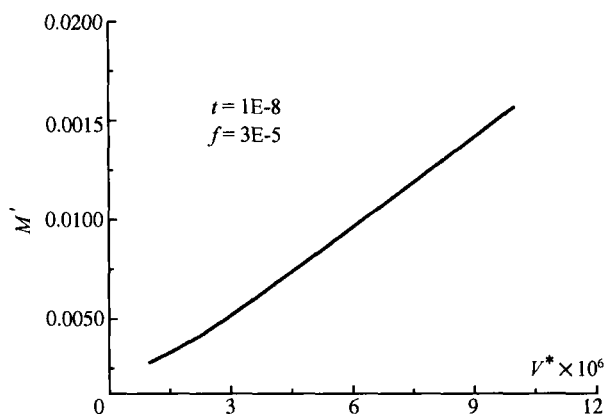


Fig. 6. Variation of mass melting rate with sliding velocity.

Figs. 6 and 7 give the variation curves of the mass melting rate obtained numerically by Eq. (16). In Fig. 6, in order to truly reflect the effect of sliding velocity, the mass melting rate is denoted by $M' = M^* V^* / (5 \times 10^{-7})$. It is clear that the melting mass rate is a monotonically increasing function of the dimensionless sliding velocity V^* and temperature difference t , which accords with the law and phenomena of heat transfer. Because the increase of the

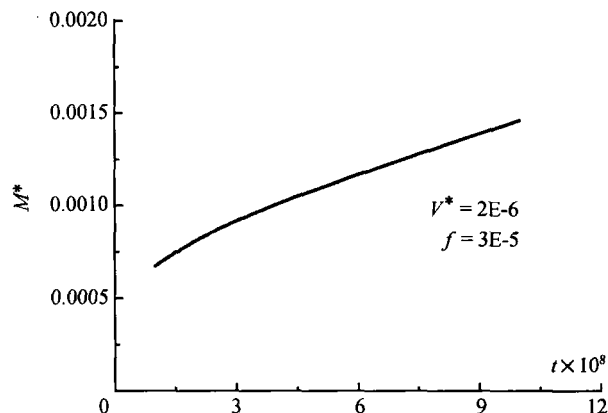


Fig. 7. Variation of mass melting rate with temperature difference.

sliding velocity will increase the friction dissipation, more PCM will melt.

3 Conclusions

The analytical model of sliding contact melting due to the temperature difference and friction is set up, and the fundamental equations for the melting process are derived. The thickness of boundary layer, the pressure distribution inside boundary layer and the mass melting rate of PCM are obtained numerically, which include the results for the contact melting driven only by the friction. It is shown that (1) the temperature difference and sliding velocity are two important effect factors, the larger the temperature difference is, the larger the thickness of boundary layer and the mass melting rate are, and the more asymmetric the pressure distribution is. The quicker the sliding velocity is, the larger the mass melting rate is, the flatter the thickness of boundary layer is, and the more symmetric the pressure distribution is; (2) the results of the contact melting driven together by the friction and temperature difference cannot be obtained by the addition of those driven respectively by the friction and temperature difference.

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